(1st child)

~~A Simple Model of the Interplanetary Magnetic Field~~, I:

Calculation of the Magnetic Field

X64-12447

39 p

X64 12446 X64 12448

Corp. auth. as main entry NASA.
David Stern
Theoretical Division
Goddard Space Flight Center,
Greenbelt, Maryland 20771

Code 2A

TMX 54578

12447

Abstract

A simple model of the interplanetary magnetic field is described and solved analytically. In this model, space is divided into three regions by two concentric spheres, conductivities (with one exception) are assumed to be isotropic and constant in each region and flow velocities are regular and prescribed. The innermost region rotates rigidly around its center, the intermediate region contains a compressible fluid flowing radially outward with constant velocity (an idealization of the solar wind) and the outermost region is at rest. The magnetic field originates in point sources at the origin and possibly also in a uniform field at infinity. Methods are described for finding the field under these assumptions, in the general case and also in the limit when all conductivities are very high. As an example, the case in which the field's source is a point dipole, aligned with the axis of rotation, is solved in some detail.

author

AUTHOR

In its A Simple Model of the Interplanet. Magnetic Field [1963] 25 p refs

~~Available to NASA Offices~~ and ~~NASA Centers Only.~~

(See X64-12446 07-12)

AS

Introduction

Experimental evidence available now indicates the existence of an interplanetary magnetic field originating at the sun and extending at least to the earth's orbit, possibly much further. Many features of this field are still uncertain, but two of its main properties have been predicted theoretically and seem to be, so far, in agreement with observation. Both may be regarded as manifestations of the fact that a highly conducting fluid - here, the solar wind emanating radially from the sun - tends to impart its motion to lines of force embedded in it. First, it was predicted that the solar wind will stretch the lines of force, rendering them almost radial and causing the field intensity B to fall off less rapidly than it would otherwise (e.g. Alfvén, 1956). Secondly, in addition to radial stretching, the field was expected to be twisted by solar rotation into an archimedean spiral. This point was noted first by Chapman (1928) who observed that the locus of a particle stream constantly emitted from a point on the sun is, at any time, such a spiral (the same locus is described by droplets from a rotating sprinkler, for which reason the above is sometimes called the "garden-hose effect"). A line of force drawn out by a stream of particles would also follow such a spiral, and it was argued that similar twisting occurs in any field originating in the rotating sun. Parker (1958) gave a formal proof of this, assuming that the magnetic field is parallel to the velocity field as seen from a frame of reference co-rotating with the sun. The effect has also been deduced from experimental data, from the arrival direction of solar flare particles (McCracken, 1962) and from direct observation by Mariner II (Snyder, 1963). The "garden-hose angle" between B and the radial direction from the sun was there of the order of 45°.

In this work, a simple model of the interplanetary magnetic field will be investigated, first in the limiting case of a perfectly conducting fluid and then for finite, isotropic and homogeneous conductivity. The model is as follows:

Let space be divided into three regions by two concentric spheres of radii R_0 and R_1 (fig.1).Region I , the innermost,is assumed to rotate rigidly with angular velocity ω .This region also contains the source of the magnetic field,which will be assumed to be concentrated at the origin.Because of the tedious calculations involved, only the case of a dipole source with moment $\underline{M}$ will be treated in any detail.If this dipole is inclined with an angle κ to the axis of solar rotation,one may regard its axial component as an idealisation of the main solar field and its equatorial component as that of an active region co-rotating with the sun.Region II, between the spheres,contains a compressible conducting fluid flowing out radially with constant velocity u . Finally,in region III which extends to infinity,no motion takes place. Region I here represents the sun,region II the space swept by the solar wind.The equation connecting the magnetic field $\underline{B}$, the electric field $\underline{E}$, the velocity $\underline{v}$ and the conductivity σ is

$$\text{curl } \underline{B} = \mu_0 \sigma \left[\underline{E} + (\underline{v} \times \underline{B}) \right] \tag{1}$$

and we shall be interested in stationary solutions,with particular interest in the case when σ is large and R_1 considerably in excess of R_0 .

It should be borne in mind all along that the preceding is a gross oversimplification of the actual situation.To stress this point,all approximations and neglections used will now be listed.

(1) It is not established that the solar dipole plays a major role in creating the interplanetary field.Certainly,the source of the field is quite complex.

(2) The solar wind is not an ordinary conducting fluid but a nearly collisionless plasma,conducting very well along the magnetic field but much less across it. Unfortunately,since the direction of the conduction anisotropy depends on the magnetic field,taking it into account makes the equation of conduction nonlinear. Except for one case,therefore,the conductivity will be assumed to be isotropic.

(3) The flow and field assumed here are laminar and regular,while observation

indicates a large irregular component. The model developed here thus represents only the effects of the average interplanetary field and does not include the turbulent component.

(4) The assumed sharp boundaries are only a convenient approximation to the actual ones, which are not too well known at the present time.

(5) In any problem of this sort, the velocity $\underline{v}$ is generally not determined a priori but has to be solved simultaneously with $\underline{B}$, using the hydromagnetic flow equation (e.g. Chandrasekar [1956]). This equation is nonlinear and one notices that with this approach the term $(\underline{v} \times \underline{B})$ in (1) becomes nonlinear too. In the vicinity of the earth, of course, the mass flow dictates the magnetic field because of its much higher energy density ; nevertheless, in the vicinity of what corresponds to the outer sphere in this model, the flow may be considerably distorted by the field.

Unfortunately, a more realistic model would be very hard to solve analytically. It is hoped, however, that the results obtained here will give some qualitative insight about the behavior of the actual interplanetary field.

Infinite Conductivity

The field produced when $\sigma \to \infty$ has been derived by Parker (1958), utilizing a rotating frame of reference. It will be derived here in a somewhat more conventional way.

If the conductivity tends to infinity and $\underline{B}$ does not, in general

$$\underline{E} = -[\underline{v} \times \underline{B}] \qquad (2)$$

Taking the curl in region II gives, in spherical coordinates (r, ϑ, φ)

$$\frac{\partial \underline{B}}{\partial t} = u \operatorname{curl} [\underline{i}_\varphi B_\vartheta - \underline{i}_\vartheta B_\varphi] \qquad (3)$$

Utilizing

$$\operatorname{div} \underline{B} = 0 \qquad (4)$$

one obtains

$$\frac{\partial X}{\partial t} - u\frac{\partial X}{\partial r} = 0 \tag{5}$$

where X stands for either r^2B_r , $r\,B_\varphi$ or $r\,B_\vartheta$ .Under rotational symmetry X is independent of time and one obtains

$$B_r = \xi(\vartheta)\,r^{-2} \tag{6a}$$

$$B_\varphi = \zeta(\vartheta)\,r^{-1} \tag{6b}$$

and with (4)

$$B_\vartheta = 0 \tag{6c}$$

On the surface $r = R_0$, E_ϑ is continuous.Just inside the boundary,by (2)

$$E_\vartheta = -\omega R_0 \sin\vartheta\, B_r$$

and ,because B_r is continuous,its value just outside the boundary may be used. The continuity of E_ϑ then gives

$$\zeta(\vartheta) = -(\omega/u)\sin\vartheta\;\xi(\vartheta) \tag{7}$$

so that the tangent of the "garden-hose angle" is

$$\operatorname{tg}\chi = B_\varphi/B_r = -(\omega/u)\,r\sin\vartheta \tag{8}$$

In general,the source of the field rotates

$$B(r,\vartheta,\varphi,t) = B(r,\vartheta,\lambda) \tag{9a}$$

$$\lambda = \varphi - \omega t \tag{9b}$$

$$\frac{\partial}{\partial t} = -\omega\frac{\partial}{\partial\varphi} \tag{9c}$$

then (5) becomes

$$\frac{\partial X}{\partial\lambda} - \frac{u}{\omega}\frac{\partial X}{\partial r} = 0 \tag{10}$$

and its solution is

$$X = X(\vartheta, \varphi - \omega t - \omega r/u) \tag{11}$$

Thus

$$B_r = r^{-2} \sum_m \xi_m(\vartheta) \exp im(\varphi - \omega t - \omega r/u) \tag{12a}$$

and the continuity of E_ϑ on $r = R_0$ gives

$$B_\varphi = -(\omega/ur)\sin\vartheta \sum_m \xi_m(\vartheta) \exp im(\varphi - \omega t - \omega r/u) \tag{12b}$$

For a more detailed solution it is generally better to solve (1) for finite conductivity and then investigate the behavior of the solution when σ gets large.

Finite Conductivity

We now turn to solving the problem for arbitrary σ. It will be assumed that σ is uniform in each region and takes the values σ_1, σ_2 and σ_3 in regions I, II and III respectively. The following theorem is found useful : if a vector field $\underline{B}$ satisfies (4), it may be uniquely resolved in the following manner.

$$\underline{B} = \operatorname{curl} \Psi_1 \underline{r} + \operatorname{curl}\operatorname{curl} \Psi_2 \underline{r} \tag{13}$$

Following Elsasser (1946) the component fields will be termed the toroidal and poloidal components, respectively. The theorem was first proved for rotational symmetry by Lüst & Schlüter (1954) and for the general case by Backus (1958). The following identities hold generally (Smythe, 1950)

$$\operatorname{curl} \Psi \underline{r} \equiv [\operatorname{grad} \Psi \times \underline{r}] \tag{14a}$$

$$\operatorname{curl}\operatorname{curl} \Psi \underline{r} \equiv \operatorname{grad} \frac{\partial}{\partial r}(\Psi r) - \underline{r}\, \nabla^2 \Psi \tag{14b}$$

Using (13), a vector potential may be defined

$$\underline{A} = \Psi_1 \underline{r} + \operatorname{curl} \Psi_2 \underline{r} \tag{15a}$$

$$\underline{B} = \operatorname{curl} \underline{A} \tag{15b}$$

$\underline{A}$ satisfies the gauge condition

$$\operatorname{div} \underline{A} = r^{-2} \frac{\partial}{\partial r}(\Psi_1 r^3)$$

and the electric field $\underline{E}$ may be expressed

$$\underline{E} = -\operatorname{grad} \Psi_0 - \frac{\partial \underline{A}}{\partial t} \tag{16}$$

The problem thus reduces to solving (1) for three unknown scalars Ψ_0, Ψ_1 and Ψ_2. The derivation of a general solution tends to be tedious and will therefore only be outlined in this section. In each of the three regions, equation (1) can be brought to the form

$$\operatorname{grad} \xi_i + \underline{r} \eta_i + \operatorname{curl} \underline{r} \zeta_i = 0 \tag{17}$$

(i is the region's index). Taking the curl and applying the uniqueness of the resolution (13) gives

$$\operatorname{curl} \underline{r} \eta_i = \operatorname{curl} \operatorname{curl} \underline{r} \zeta_i = 0$$

from which

$$\eta_i = \eta_i(r)$$

$$\zeta_i = \zeta_i(r)$$

and after substitution in (17)

$$\xi_i = \xi_i(r)$$

The expressions η_i and ζ_i will in general be functions of Ψ_1 and Ψ_2, which are also defined within an arbitrary function of r. If these functions can be chosen so that η_i vanishes, equation (17) gives

$$\xi_i = \text{const} = C_i$$

<u>Region I</u>

In this region

$$\underline{v} = (\underline{\omega} \times \underline{r}) = \underline{i}_\varphi \, \omega r \sin\vartheta \tag{18}$$

the following relations hold for $\underline{v}$ and for any vector $\underline{A}$

$$\underline{v} = \tfrac{1}{2} \operatorname{curl}(\underline{\omega} r^2)$$

$$\operatorname{curl} \underline{v} = 2\underline{\omega}$$

$$(\underline{A}\, \underline{\nabla})\, \underline{v} = (\underline{\omega} \times \underline{A})$$

$$(\underline{v}\, \underline{\nabla})\, \underline{A} = (\omega \times A) + \omega\left(\underline{i}_r \frac{\partial A_r}{\partial \varphi} + \underline{i}_\vartheta \frac{\partial A_\vartheta}{\partial \varphi} + \underline{i}_\varphi \frac{\partial A_\varphi}{\partial \varphi} \right)$$

(e.g. see Morse and Feshbach [1953] eq.1.4.3)

To resolve $(\underline{v} \times \underline{B})$ in the manner of eq.(17) one notes

$$(\underline{v} \times \underline{B}) = (\underline{v} \times \operatorname{curl} \underline{A})$$

$$= \operatorname{grad}(\underline{A}\underline{v}) - (\underline{A} \times \operatorname{curl} \underline{v}) - (\underline{A}\underline{\nabla})\underline{v} - (\underline{v}\underline{\nabla})\underline{A}$$

$$= \operatorname{grad}(\underline{A}\underline{v}) - \omega\left(\underline{i}_r \frac{\partial A_r}{\partial \varphi} + \underline{i}_\vartheta \frac{\partial A_\vartheta}{\partial \varphi} + \underline{i}_\varphi \frac{\partial A_\varphi}{\partial \varphi}\right)$$

$$(\underline{v} \times \underline{B}) = -\operatorname{grad}\left(\omega r \sin\vartheta \frac{\partial \Psi_2}{\partial \vartheta}\right) - \underline{r}\,\omega \frac{\partial \Psi_1}{\partial t} - \operatorname{curl}\left(\underline{r}\,\omega \frac{\partial \Psi_2}{\partial t}\right) \tag{19}$$

The rest of (1) is easily resolved in the prescribed manner using (13),(15) and (16),giving

$$\xi_1(r) = \frac{\partial}{\partial r}(\Psi_1 r) + \mu_0\sigma_1\Psi_0 + \mu_0\sigma_1\omega r \sin\vartheta \frac{\partial \Psi_2}{\partial \vartheta} \tag{20a}$$

$$\eta_1(r) = \mu_0\sigma_1\left(\frac{\partial \Psi_1}{\partial t} + \omega \frac{\partial \Psi_1}{\partial \varphi}\right) - \nabla^2\Psi_1 \tag{20b}$$

$$\zeta_1(r) = \mu_0\sigma_1\left(\frac{\partial \Psi_2}{\partial t} + \omega \frac{\partial \Psi_2}{\partial \varphi}\right) - \nabla^2\Psi_2 \tag{20c}$$

The option to add an arbitrary radial function to Ψ_1 and Ψ_2 is used to make both $\eta_1(r)$ and $\zeta_1(r)$ vanish,so that

$$\frac{\partial}{\partial r}(\Psi_1 r) + \mu_0\sigma_1\Psi_0 + \mu_0\sigma_1\omega r \sin\vartheta \frac{\partial \Psi_2}{\partial \vartheta} = C_1 \tag{21}$$

So far no use has been made of the fact that the field's source co-rotates with region I,so that the equations hold even when the field originates,say,in region III as a fixed "interstellar field".If equations (9) are now introduced, one obtains at once

$$\nabla^2\Psi_1 = 0 \tag{22a}$$

$$\nabla^2\Psi_2 = 0 \tag{22b}$$

If one assumes the field's source is concentrated at the origin,it is useful to expand Ψ_1 and Ψ_2 in spherical harmonics; the expansion of Ψ_2 then has a singularity at the origin corresponding to the source of the field.For instance, if this source is a dipole with moment $\underline{M}$, inclined at an angle κ to the rotation axis

$$\Psi_1 = \sum_{nm} a_{nm} \rho^n P_n^m(\vartheta) \exp im(\varphi - \omega t) \tag{23a}$$

$$\Psi_2 = \sum_{nm} b_{nm} \rho^n P_n^m(\vartheta) \exp im(\varphi - \omega t)$$
$$+ \beta \rho^{-2} \left\{ \cos k\, P_1^0(\vartheta) + \sin k\, P_1^1(\theta) \cos(\varphi - \omega t) \right\} \tag{23b}$$

where

$$\beta = \mu_0 M / 4\pi R_0^2$$

Region II

In this region

$$\underline{v} = u\, \underline{i}_r$$

The contribution of the toroidal component to ($\underline{v} \times \underline{B}$) is

$$u\,(\underline{i}_r \times \operatorname{curl} \underline{i}_r \Psi_1 r) = u\,(\underline{i}_r \times (\operatorname{grad} \Psi_1 r \times \underline{i}_r))$$
$$= u\,\left\{ \operatorname{grad}(\Psi_1 r) - \underline{i}_r \frac{\partial}{\partial r}(\Psi_1 r) \right\} \tag{24}$$

and of the poloidal component, by (14a)

$$u\,(\underline{i}_r \times \operatorname{grad} \frac{\partial}{\partial r}(\Psi_2 r)) = -\operatorname{curl} \underline{r} \left(\frac{u}{r} \frac{\partial}{\partial r}(\Psi_2 r) \right) \tag{25}$$

All other terms of (1) are the same as in region I, therefore

$$\xi_2(r) = \frac{\partial}{\partial r}(\Psi_1 r) + \mu_0 \sigma_2 \Psi_0 - u \mu_0 \sigma_2 \Psi_1 r \tag{26a}$$

$$\eta_2(r) = \mu_0 \sigma_2 \left(\frac{\partial \Psi_1}{\partial t} + \frac{u}{r} \frac{\partial}{\partial r}(\Psi_1 r) \right) - \nabla^2 \Psi_1 \tag{26b}$$

$$\zeta_2(r) = \mu_0 \sigma_2 \left(\frac{\partial \Psi_2}{\partial t} + \frac{u}{r} \frac{\partial}{\partial r}(\Psi_2 r) \right) - \nabla^2 \Psi_2 \tag{26c}$$

As in region I, Ψ_1 and Ψ_2 are chosen so that η_2 and ζ_2 vanish and ξ_2 equals a constant C_2. Using (9), the angular part of Ψ_1 is now expanded in spherical harmonics

$$\Psi_1 = \sum_{nm} G_{nm}(r) P_n^m(\vartheta) \exp im(\varphi - \omega t) \tag{27}$$

Inserting into (26b) and using the independence of spherical harmonics gives, for any n and m

$$\frac{1}{r} \frac{d^2}{dr^2}(r G_{nm}) - \frac{u \mu_0 \sigma_2}{r} \frac{d}{dr}(r G_{nm}) - \left(\frac{n(n+1)}{r^2} - im\omega \mu_0 \sigma_2 \right) G_{nm} = 0 \tag{28}$$

Multiplying by $r R_0^2$, introducing the variable $\rho = r/R_0$ and defining

$$y_{nm}(\varrho) = \varrho\, G_{nm}(\varrho)$$

gives

$$y''_{nm} - u\mu_0\sigma_2 R_0\, y'_{nm} + (i\mu_0\sigma_2\omega R_0^2 m - n(n+1)\varrho^{-2})\, y_{nm} = 0 \qquad (29)$$

Let the magnetic Reynolds number associated with radial outflow (in region II) be defined

$$2\alpha = \mu_0\sigma_2 R_0 u$$

and that associated with rotation

$$2\alpha_{2\omega} = \mu_0\sigma_2 R_0^2\omega$$

substituting

$$y_{nm} = u_{nm}\, \exp(\alpha\varrho)$$

and defining

$$\alpha_{2c}^2 = \alpha^2 - 2im\alpha_{2\omega}$$

$$\alpha_{2c} = \alpha_{2r} + i\alpha_{2i}$$

($\alpha_{2r}(m)$ and $\alpha_{2i}(m)$ real, α_{2m} positive) this becomes

$$u''_{nm} - u_{nm}\left(\alpha_{2c}^2 + n(n+1)\varrho^{-2}\right) = 0 \qquad (30)$$

Let operators $\mathcal{L}_n$ be defined

$$\mathcal{L}_n = \varrho^{-(n+1)}\left[\varrho^3 \frac{d}{d\varrho}\right]^n \varrho^{-(2n-1)} \qquad (31)$$

Then the general solution of (32) may be written ([Murphy,1960] ; p.337,eq.256)

$$u_{nm} = A_1\, \mathcal{L}_n[\exp(\alpha_{2c}\varrho)] + A_2\, \mathcal{L}_n[\exp(-\alpha_{2c}\varrho)]$$

$$= A_1\, u_{nm1}(\varrho) + A_2\, (-1)^n\, u_{nm2}(\varrho) \qquad (32)$$

Defining (i = 1,2)

$$g_{nmi} = \varrho^{-1}\exp(\alpha\varrho)\, u_{nmi}$$

the general form of Ψ_1 in region II may be written

$$\Psi_1 = \sum_{nm} P_n^m(\theta)\exp im(\varphi-\omega t)\left[a_{nm1}\, g_{nm1}(\varrho) + a_{nm2}\, g_{nm2}(\varrho)\right] \qquad (33a)$$

and that of Ψ_2

$$\Psi_2 = \sum_{nm} P_n^m(\theta)\exp im(\varphi-\omega t)\left[b_{nm1}\, g_{nm1}(\varrho) + a_{nm2}\, g_{nm2}(\varrho)\right] \qquad (33b)$$

<u>Region III</u>

In this region (1) has no ($\underline{v} \times \underline{B}$) term. Consequently

$$\xi_3(r) = \frac{\partial}{\partial r}(\Psi, r) + \mu_0 \sigma_3 \Psi_0 \tag{34a}$$

$$\eta_3(r) = \mu_0 \sigma_3 \frac{\partial \Psi_1}{\partial t} - \nabla^2 \Psi_1 \tag{34b}$$

$$\zeta_3(r) = \mu_0 \sigma_3 \frac{\partial \Psi_2}{\partial t} - \nabla^2 \Psi_2 \tag{34c}$$

As before, Ψ_1 and Ψ_2 are chosen so that η_3 and ζ_3 vanish and ξ_3 is a constant C_3. Let Ψ_1 be expanded in a fashion similar to (27)

$$\Psi_1 = \sum_{nm} H_{nm}(r)\, P_n^m(\vartheta) \exp im(\varphi - \omega t) \tag{35}$$

If $m = 0$, the terms have no time dependence, equation (34b) shows they are harmonic and $H_{nm}(r)$ is proportional to $r^{-(n+1)}$. If $m \neq 0$, one proceeds as in region II. Defining

$$z_{nm}(\rho) = \rho\, H_{nm}(\rho)$$

and a rotational magnetic Reynolds number for region III

$$2\alpha_{3\omega} = \mu_0 \sigma_3 R_0^2 \omega$$

one obtains

$$z''_{nm} - z_{nm}\left(n(n+1)\rho^{-2} - 2im\alpha_{3\omega}\right) = 0 \tag{36}$$

which has the same form as equation (30). As was done there, one defines

$$\alpha_{3c}^2 = -2im\alpha_{3\omega}$$

$$\alpha_{3c} = \alpha_{3r} + i\alpha_{3i}$$

leading to

$$\Psi_1 = \sum P_n^m(\vartheta) \exp im(\varphi - \omega t)\left[A_{nm1} h_{nm1}(\rho) + A_{nm2} h_{nm2}(\rho)\right] \tag{37}$$

where

$$h_{nm1}(\rho) = \rho^{-1} \mathcal{L}_n[\exp(\alpha_{3c}\rho)]$$

$$h_{nm2}(\rho) = (-1)^n \rho^{-1} \mathcal{L}_n[\exp(-\alpha_{3c}\rho)]$$

Choosing α_{3r} as positive, one finds that $h_{nm1}(\rho)$ contains an exponential exp $(\alpha_{3r}\rho)$ which causes it to diverge. Therefore, all A_{nm1} vanish and (37) becomes

$$\Psi_1 = \sum A_{nm2}\, h_{nm2}(\rho)\, P_n^m(\vartheta)\, \exp im(\varphi - \omega t) \tag{38}$$

In a similar manner

$$\Psi_2 = \sum B_{nm2}\, h_{nm2}(\rho)\, P_n^m(\vartheta)\, \exp im(\varphi - \omega t) \tag{39}$$

In (39), contributions by a fixed outside source may be included. Usually, equations (9) then no longer hold and the calculation is somewhat different.

Boundary Conditions

On the boundaries the components of $\underline{B}$, the tangential one of $\underline{E}$ and the normal one of curl $\underline{B}$ are continuous. Let the operator Λ^2 be defined (Backus, 1958)

$$\Lambda^2 = \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta} + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2} \tag{40}$$

so that

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\Lambda^2 \tag{41}$$

and

$$[\Lambda^2 + n(n+1)]\, P_n^m(\vartheta)\exp im\varphi = 0 \tag{42}$$

By (13), (14b)

$$r B_r = -\Lambda^2\Psi_2 \tag{43}$$

and since all boundaries are spherical, this implies the continuity of all expansion terms of Ψ_2, as expressed in (23b), (33b) and (39). In a similar way

$$r\,(\text{curl}\ \underline{B})_r = -\Lambda^2\Psi_1$$

implies the continuity of the terms of Ψ_1. On a spherical boundary, curl $\underline{r}\Psi_1$ is also continuous, by (13) so is curl curl $\underline{r}\Psi_2$ and one may show from this that $\partial\Psi_2/\partial r$ is also continuous across the boundaries. Finally, using the above results with (15a), (16) one finds that Ψ_0 is also continuous. There are thus four conditions to be met on each of the two boundaries and eight sets of undetermined coefficients. Because the source of the field is poloidal, the continuity of the components of Ψ_2 and $\partial\Psi_2/\partial r$ make it possible to evaluate Ψ_2 independently. Next Ψ_1 is found, using (21), (26a) and (34a) to express the continuity of Ψ_0. Finally Ψ_0 is obtained, using the three equations mentioned above.

As σ_1 and σ_2 increase without limit, the solutions tend to approach those obtained before by assuming $\underline{E} = -(\underline{v} \times \underline{B})$. In general, the solutions in region III diverge unless σ_3 is kept finite. This may be interpreted as meaning that in a system having stationary and rotating ideal conductors in contact, infinite currents will be excited by unipolar induction. If the hydromagnetic equation were used, this divergence would not occur because infinite forces would act on the flow and distort its pattern.

In the worked example that follows this divergence is avoided in a different way, namely by the assumption that in region III conductivity is arbitrary (finite or infinite) along lines of force and zero across them.

A Worked Example

As a simple illustration of the preceding, the case when the field source is a dipole of moment $\underline{M}$, located at the origin and aligned with the axis of rotation, will now be considered. This source produces an axisymmetric field and therefore the variables φ and t as well as the expansion index m are absent. It is then found convenient to use dimensionless units and to define

$$\rho = r/R_0$$

$$\Psi_{01} = \Psi_0 / u R_0$$

$$2\alpha_1 = \mu_0 \sigma_1 u R_0$$

The scalars defining the field then obey the following equations. In region I

$$\nabla^2 \Psi_1 = 0 \tag{22a}$$

$$\nabla^2 \Psi_2 = 0 \tag{22b}$$

$$\Psi_{01} = -\left(\frac{\omega}{u}\right) \rho \sin\vartheta \frac{\partial \Psi_2}{\partial \vartheta} - \frac{1}{2\alpha_1} \frac{\partial}{\partial \rho}(\Psi_1 \rho) + C_{11} \tag{21}$$

in II

$$\nabla^2 \Psi_1 - \frac{2\alpha}{\rho} \frac{\partial}{\partial \rho}(\Psi_1 \rho) = 0 \tag{44a}$$

$$\nabla^2 \Psi_2 - \frac{2\alpha}{\rho} \frac{\partial}{\partial \rho}(\Psi_2 \rho) = 0 \tag{44b}$$

$$\Psi_{01} = \Psi_1 \rho - \frac{1}{2\alpha} \frac{\partial}{\partial \rho}(\Psi_1 \rho) + C_{21} \tag{44c}$$

and in III, if an isotropic conductivity σ_3 were assumed

$$\nabla^2 \Psi_1 = 0 \tag{45a}$$

$$\nabla^2 \Psi_2 = 0 \tag{45b}$$

$$\Psi_{01} = -\left(1/\mu_0 \sigma_3 u R_0\right) \frac{\partial}{\partial \rho}(\Psi_1 \rho) + C_{31} \tag{45c}$$

As was mentioned at the end of the previous section, in order to prevent divergence as $\sigma_3 \to \infty$ and also to take into account, at least partially, the anisotropy of conductivity, it will be assumed that in region III the conductivity is σ_3 for flow along lines of force and zero for flow across them. Then, in region III

$$\underline{B} \times \text{curl } \underline{B} = 0 \tag{46a}$$

$$\underline{B} \cdot \text{curl } \underline{B} = \mu_0 \sigma_3 (\underline{E} \cdot \underline{B}) \tag{46b}$$

These equations are nonlinear ; however, it will be shown that a solution having the required form near the boundary is obtained by taking

$$\text{curl } \underline{B} = 0 \tag{46c}$$

from which

$$(\underline{E} \cdot \underline{B}) = 0 \tag{46d}$$

Using (13), (14b) and applying the treatment of (17), equation (46c) gives

$$\nabla^2 \Psi_2 = 0 \tag{47a}$$

$$\Psi_1 = 0 \tag{47b}$$

The unknown scalars are expanded :

In I, as in (23)

$$\Psi_1 = \sum a_n \rho^n P_n(\vartheta) \tag{48a}$$

$$\Psi_2 = \sum b_n \rho^n P_n(\vartheta) + \beta \rho^{-2} \cos\vartheta \tag{48b}$$

In II

$$\Psi_1 = \sum [a_{n1} g_{n1}(\rho) + a_{n2} g_{n2}(\rho)] P_n(\vartheta) \tag{49a}$$

$$\Psi_2 = \sum [b_{n1} g_{n1}(\rho) + b_{n2} g_{n2}(\rho)] P_n(\vartheta) \tag{49b}$$

and in III

$$\Psi_2 = \sum B_n (\rho_1/\rho)^{n+1} P_n(\vartheta) \tag{50}$$

To begin with, Ψ_2 is derived. For any $n \neq 1$, the continuity equations of Ψ_2 and $\partial\Psi_2/\partial r$ yield a set of 4 equations with unknowns b_n, b_{n1}, b_{n2} and B_n. Because of the form of the source term in (48b) these equations are homogeneous and all coefficients vanish. For $n = 1$, by (31)

$$g_{11}(\rho) = (\alpha\rho^{-1} - \rho^{-2}) \exp 2\alpha\rho \tag{51a}$$

$$g_{12}(\rho) = \alpha\rho^{-1} + \rho^{-2} \tag{51b}$$

Denoting differentiation with respect to ρ by dashes and defining $\rho_1 = R_1 / R_0$

$$\begin{aligned} b_1 + \beta &= b_{11}\, g_{11}(1) + b_{12}\, g_{12}(1) \\ b_1 - \beta &= b_{11}\, g'_{11}(1) + b_{12}\, g'_{12}(1) \\ B_1 &= b_{11}\, g_{11}(\rho_1) + b_{12}\, g_{12}(\rho_1) \\ -2B_1 &= b_{11}\rho_1 g'_{11}(\rho_1) + b_{12}\rho_1 g'_{12}(\rho_1) \end{aligned} \tag{52}$$

The complete solution is rather lengthy. If $\alpha \gg 1$, which represents the "high conductivity" case, and $\rho_1 \gg 1$, one obtains approximately in I

$$\Psi_2 = \beta \cos\vartheta \left(\tfrac{1}{2}\rho + \rho^{-2}\right) \tag{53a}$$

in II

$$\Psi_2 = (3\beta/2\rho) \cos\vartheta \left[1 - (2\alpha\rho_1)^{-1} \exp -2\alpha(\rho_1 - \rho)\right] \tag{53b}$$

and in III

$$\Psi_2 = (2\beta\rho_1/2\rho^2) \cos\vartheta \tag{53c}$$

The poloidal field in III is thus that of an axial dipole while in II

$$B_r = (3\beta/R_0) \cos\vartheta\, \rho^{-2} \tag{54a}$$

$$B_\vartheta = (3\beta/2R_0\rho_1) \sin\vartheta \exp -2\alpha(\rho_1 - \rho) \tag{54b}$$

which may be compared to (6) . In I

$$B_r = (\beta/R_0) \cos\vartheta\, (2\rho^{-3} + 1)$$

$$B_\vartheta = (\beta/R_0) \sin\vartheta\, (\rho^{-3} - 1)$$

The lines of force of this solution are given in fig.2, in the limit of high conductivity.

Because of (47b), Ψ_1 is expressed by only three sets of undetermined coefficients and the continuity of Ψ_0 on $r = R_1$ does not have to be invoked. By (21), in I

$$\Psi_{01} = \frac{\omega}{u}\left(b_1 \rho^2 + \beta/\rho \right) \sin^2\vartheta - \frac{1}{2\alpha_1} \frac{\partial}{\partial \rho}(\Psi_1 \rho) + c_{11} \tag{55}$$

Because of

$$\sin^2\vartheta \equiv (2/3)(P_0 - P_2) \tag{56}$$

only for $n = 0,2$ do the coefficients of Ψ_1 satisfy inhomogeneous equations and therefore differ from zero. The monopole terms do not contribute to the magnetic field, so that only the quadrupole part has to be evaluated. One finds, in region II

$$g_{21}(\rho) = (\alpha^2 \rho^{-1} - 3\alpha\rho^{-2} + 3\rho^{-2}) \exp 2\alpha\rho \tag{57a}$$

$$g_{22}(\rho) = \alpha^2 \rho^{-1} + 3\alpha\rho^{-2} + 3\rho^{-3} \tag{57b}$$

from which

$$\begin{aligned} a_2 &= a_{21} g_{21}(1) + a_{22} g_{22}(1) \\ 0 &= a_{21} g_{21}(\rho_1) + a_{22} g_{22}(\rho_1) \\ 2\alpha_1 \gamma - 3 a_2 &= a_{21} \lambda_1 + a_{22} \lambda_2 \end{aligned} \tag{58}$$

where

$$\gamma = -(2\omega/3u)(b_1 + \beta)$$

$$\lambda_1 = -3\alpha_1 (1 - 2/\alpha) \exp 2\alpha$$

$$\lambda_2 = 2\alpha_1 (\alpha^2 + 3\alpha + 9/2 + 3\alpha^{-1})$$

This again tends to produce cumbersome expressions. When $\alpha \gg 1$, $\rho_1 \gg 1$, one obtains in II

$$\Psi_1 \cong \gamma \rho^{-1} \left[1 - \exp 2\alpha(\rho - \rho_1) \right] P_2(\vartheta) + f(r) \tag{59a}$$

and in I

$$\Psi_1 \cong \gamma \rho \, P_2(\vartheta) + \text{const.} \tag{59b}$$

In region II, by (14a)

$$\mathcal{B}_\varphi \cong 3\gamma\varrho^{-1}\left[1-\exp 2\alpha(\varrho-\varrho_1)\right]\sin\vartheta\cos\vartheta \tag{60}$$

which should be compared with (6b).

Finally, Ψ_0 is derived. Equation (46d) implies that in region III the lines of force of $\underline{B}$ lie on equipotentials of Ψ_0. Now $\underline{B}$ in that region was found to be a dipole field, and its lines of force are

$$\sin^2\vartheta/\varrho = \text{const.}$$

Thus, in III

$$\Psi_0(r,\vartheta) = \Psi_0(\sin^2\vartheta/\varrho) = \sum \varepsilon_n(\sin^2\vartheta/\varrho)^n \tag{61}$$

By (31) and (32), or by direct calculation of a ϑ-independent solution of (44a), the monopole term of Ψ is, in II

$$\Psi_1(\text{mono}) = a_{01}\varrho^{-1}\exp 2\alpha\varrho + a_{02}\varrho^{-1}$$

which with (44c) gives

$$\Psi_{01}(\text{mono}) = a_{02} + C_{21}$$

Assuming as before that $\alpha \gg 1$, $\varrho_1 \gg 1$, (59a) and (44c) give, in II

$$\Psi_{01} = \gamma P_2(\vartheta) + C_{22}$$

Combining this with (61) and (56) shows that

$$C_{22} = -\gamma$$

so that in II

$$\Psi_{01} = -\tfrac{3}{2}\gamma\sin^2\vartheta \tag{62}$$

and in III

$$\Psi_{01} = -\tfrac{3}{2}\gamma\sin^2\vartheta\,(\varrho_1/\varrho) \tag{63}$$

It should be noted that Ψ_{01} in II does not depend on r : the equipotentials thus tend to be (with large α and ϱ_1) cones of constant ϑ.

The potential in I is determined in the same manner. If $\alpha_1 \gg 1$, the term containing Ψ_1 in (55) may be neglected and one gets

$$\Psi_{01} = (\omega\beta/u)\left(\tfrac{1}{2}\varrho^2 + \varrho^{-1}\right)\sin^2\vartheta \tag{64}$$

The approximation involved here is the same as in (2) , so that (46d) holds in all regions and the magnetic lines of force lie on equipotential surfaces. Thus figure 2 may also be viewed as a cross section of the equipotentials of the electric field.

It is interesting to note that (in the limiting case of high conductivity) the electric potential in region II, especially near the equatorial plane, is different from that at infinity. In particular, the equatorial plane is an equipotential in which, by (64)

$$\Psi_0 = \frac{3}{2}\omega R_0 \beta \tag{65}$$

The quantity (β/R_0) , which is of the order of the field at $r = R_0$, will be taken as one gauss ; eq. (65) then gives a potential of about $2 \cdot 10^8$ volts. This result may have some connection to the modulation of cosmic radiation by the solar activity cycle which resembles that produced by an electric field (Ehmert, 1960) ; however, the value of Ψ_0 deduced here is too small by a factor 5-10, and it should be borne in mind that when the solar dipole reverses its direction, as has been observed in 1958 (Babcock, 1959), Ψ_0 is bound to reverse its sign.

To the preceding example one may add a homogeneous "interstellar magnetic field" $\underline{B}_0$ which, in order to preserve symmetry, will be assumed to be parallel to the rotation axis (for an arbitrarily directed $\underline{B}_0$ the calculation is more involved). Such a field can be represented by a poloidal potential

$$\Psi_2 = \frac{1}{2} B_0 r \cos\vartheta \tag{66}$$

so that (50) is replaced by

$$\Psi_2 = \sum B_n \left(\rho_1/\rho\right)^{n+1} P_n(\vartheta) + \frac{1}{2} B_0 R_0 \rho \cos\vartheta \tag{67}$$

The inclusion of $\underline{B}_0$ causes a term $\frac{1}{2} R_1 B_0$ to be added on the left of the last two of equations (52). When these equations are solved, it turns out that b_{12} (the important coefficient in region II) is modified by a factor

$$1 + (B_0 R_1 / 2\beta) \exp 2\alpha(1-\rho_1) \tag{68}$$

which for large α and ρ_1 will generally differ very slightly from unity. Thus Ψ_2 in region I and in most of II is only negligibly affected and so, consequently, are Ψ_1 and Ψ_0. One notes that it is quite possible for the outlying interplanetary field to be much weaker than the surrounding interstellar one. The solar wind then scoops out a cavity in the interstellar field, as has been first suggested by Davis (1955).

Acknowledgments

I am thankful to all those who reviewed this work and helped me with their comments, especially to Dr. J. W. Dungey who pointed out the effects of the continuity of the tangential electric field and to E. W. Greenstadt.

The major part of this work was performed under a National Academy of Sciences post-doctoral resident research associateship.

References

H.Alfvén : "The Sun's General Magnetic Field" , Tellus 8 , 1 ,(1956)

H.D.Babcock : "The Sun's Polar Magnetic Field" , Astrophys.J. 130 , 364 (1959)

G.Backus : "A Class of Self-Sustaining Dissipative Spherical Dynamos" , Annals of Physics 4 , 372 (1958)

S.Chandrasekar : "Axisymmetric Magnetic Fields and Fluid Motions" , Astrophys.J. 124 , 232 (1956)

Sydney Chapman : "Solar Streams of Corpuscles : Their Geometry,Absorbtion of Light,and Penetration" , Monthly Notices Roy.Astron.Soc. 89 , 456 (1928)

L.Davis : "Interplanetary Magnetic Fields and Cosmic Rays", Phys.Rev. 100 , 1440 (1955)

W.Elsasser : "Induction Effects in Terrestial Magnetism" , Phys.Rev. 69 , 106 (1946)

A.Ehmert : "Electric Field Modulation" , Space Research (COSPAR 1960), H.Kallmann Bijl editor,North Holland 1960

R.Lüst and A.Schlüter : "Kraftfreie Magnetfelder" , Zeitschrift für Astrophysik 34 , 263 (1954)

K.G.McCracken : "The Cosmic Ray Flare Effect , 3 : Deductions regarding the Interplanetary Magnetic Field" , J.Geophys.Research 67 , 447 (1962)

P.M.Morse & H.Feshbach : "Methods of Theoretical Physics" , McGraw Hill 1953

G.M.Murphy : "Ordinary Differential Equations and their Solutions" , Van Nostrand 1960

E.Parker : "Dynamics of the Interplanetary Gas and Magnetic Fields" , Astrophys.J. 128 , 664 (1958)

W.R.Smythe : "Static and Dynamic Electricity" , 2nd edition, McGraw Hill 1950

C.Snyder : "Particles and Fields in Interplanetary Space" , Transactions of the American Geophysical Union 44 , 725 (1963)

Captions for Figures

1. Division of space into three regions

2. Lines of force for the poloidal component, in the limit of very high conductivity.

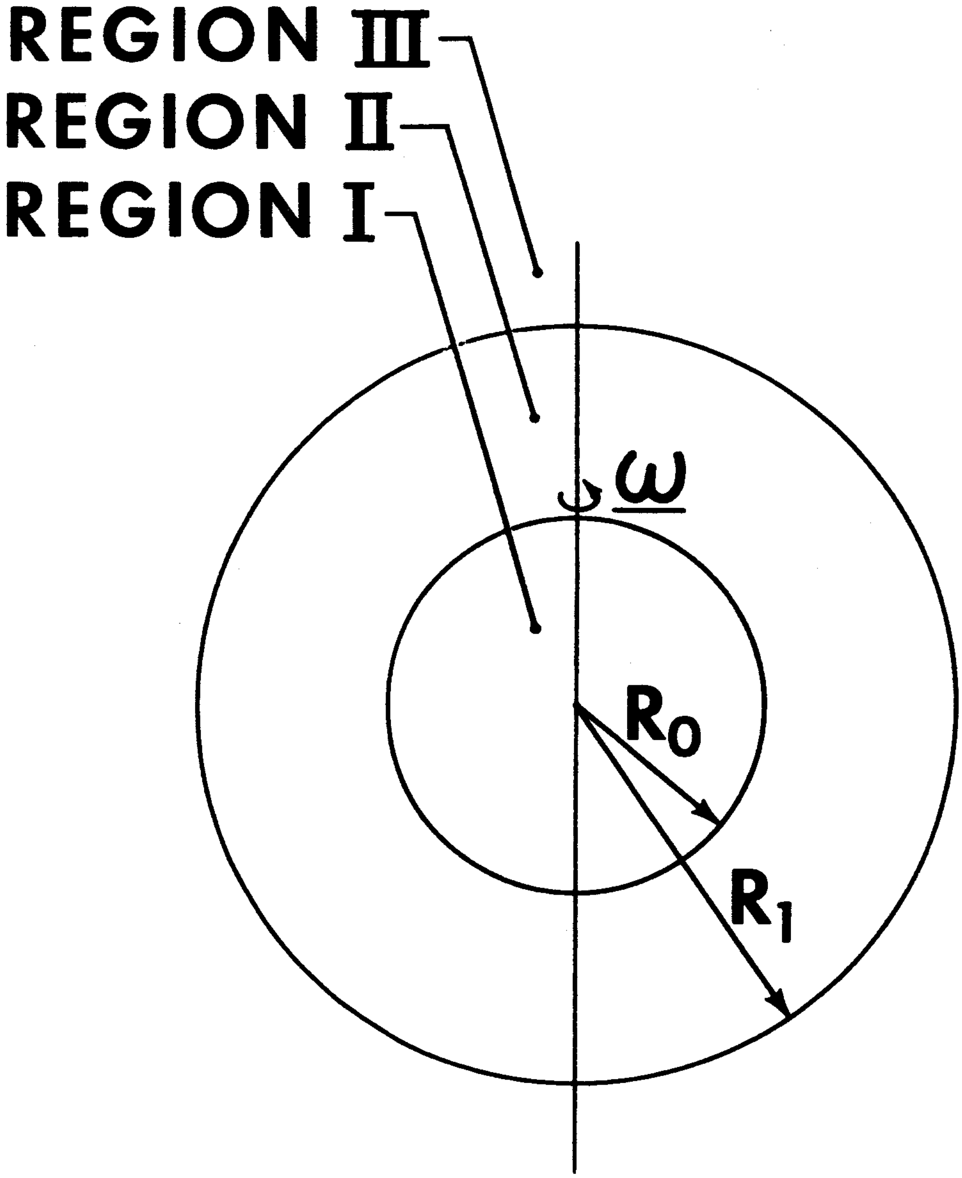

figure 1

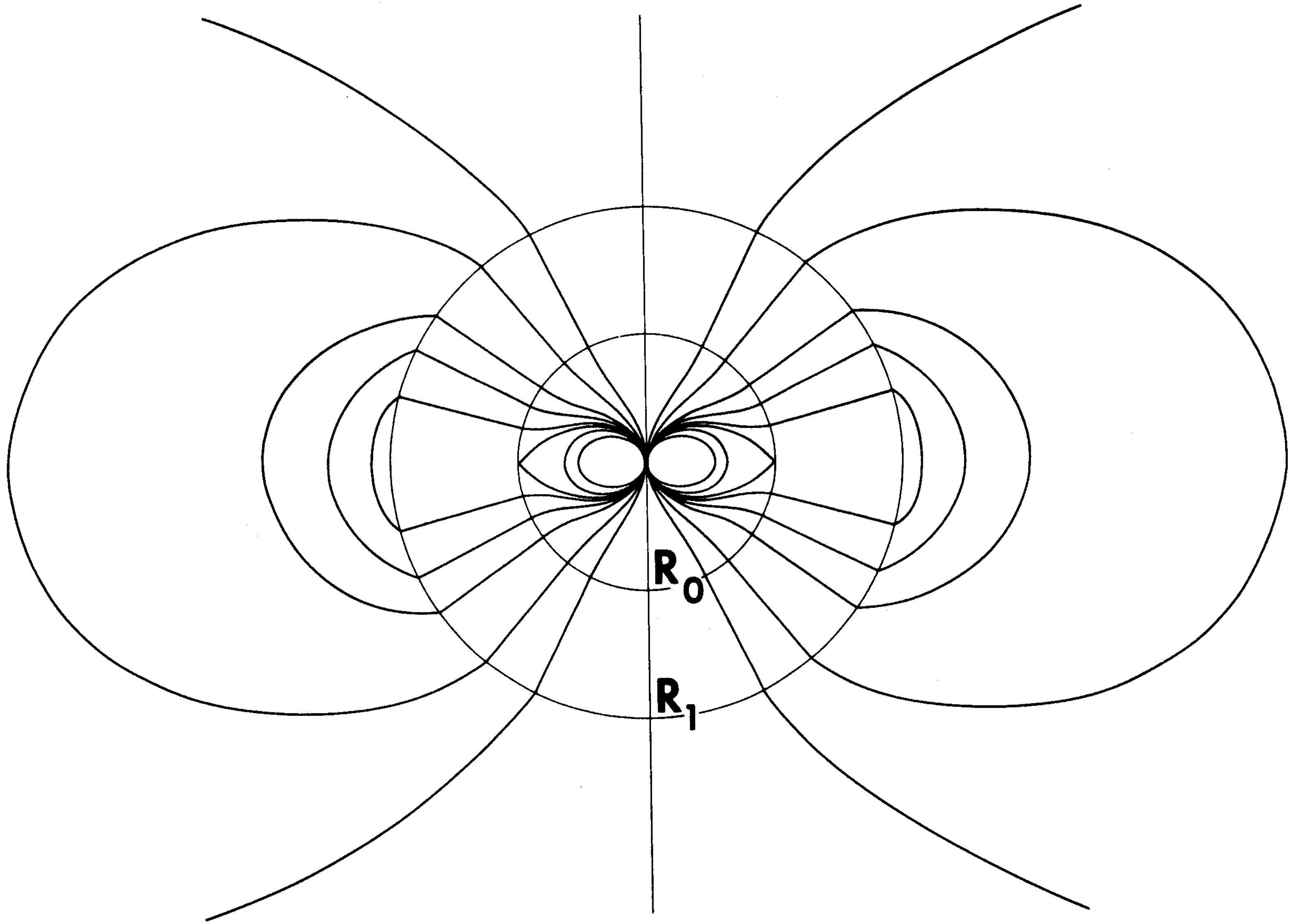

figure 2

A ~~Simple Model of the Interplanetary Magnetic Field II :~~

The Cosmic Ray Anisotropy

David Stern
Theoretical Division
Goddard Space Flight Center
Greenbelt, Maryland 20771

12448

Abstract

Three different explanations for the observed cosmic ray anisotropy are investigated. The possibility that the anisotropy is due to the existence of trapped orbits in the interplanetary magnetic field is explored by analyzing the motion of charged particles in the stretched dipole field developed in part I. It is found that an anisotropy is possible, but only when several unlikely conditions are met. Two other theories of the anisotropy, ascribing it either to a sunward flux density gradient or to the Compton-Getting effect, are then discussed. It is shown that in general both effects occur together ; for conservative fields they cancel each other and no anisotropy occurs, as might indeed be expected from Liouville's theorem. Consequently, any gradient of cosmic ray flux density which might be measured in interplanetary space is not necessarily connected with the observed anisotropy. author

AUTHOR

14 p refs

Introduction

Because of their large gyration radii in the interplanetary field, cosmic ray particles are likely to reflect in their behaviour the gross structure of this field rather than local irregularities.One of the important properties of these particles is their anisotropy,which manifests itself in a solar daily variation of about 0.3% observed on the surface of the earth.The direction of the maximum flux is approximately tangential to the earth's orbit on the afternoon side and the range of energies at which the anisotropy has been observed is roughly 7 - 20 Bev.In this range the relative modulation is nearly independent of energy (McCracken,Rao and Venkatesan,1963) , in sharp contrast with other types of cosmic ray intensity variations , which generally decrease rapidly with increasing energy.

Various theories have been advanced to explain this anisotropy,most of which fall into one of the following three classes

(1) Theories based on the Störmer effect.According to these,the anisotropy is caused by a solar-centered magnetic field in a manner similar to that by which the terrestial dipole field creates an east-west anisotropy in cosmic radiation observed near the earth's surface.

(2) Theories base on a gradient of cosmic ray flux density in the direction of the sun.

(3) Theories based on the Compton-Getting effect.In these,it is assumed that the radiation is isotropic in some frame of reference moving relative to the earth.

In what follows,these three approaches (in the above order) will be investigated in more detail.

Störmer effect theories

It has been often suggested that the cosmic ray anisotropy is caused by an interplanetary dipole field (Janossy,1937 ; Alfvén,1947 ; Dwight,1950 ; Elliot,1960,1962) . The idea is,essentially,that a weak scattering mechanism operates to fill the trapped orbits which,however,are less densely populated than free ones due to some additional loss mechanism,e.g. scattering into orbits hitting the sun (Elliot,1960).In the energy range where some of the radiation received on earth is trapped and the rest arrives directly an anisotropy will be observed,with maximum effect in the direction normal to the planes of the lines of force.

In view of the stretching and twisting expected of the (average) interplanetary field,this model has to be modified considerably.Accordingly, the motion of charged particles in the stretched dipole field,developed as a worked example in part I , will now be investigated.

The lagrangian for the motion of a particle with mass m and charge q in an electromagnetic field with magnetic and electric potentials $\underline{A}$ and Ψ_0 is

$$L = -mc^2(1 - v^2/c^2)^{1/2} + q(\underline{A}\cdot\underline{v}) - q\Psi_0 \qquad (1)$$

If neither $\underline{A}$ nor Ψ_0 depend on the azimuth angle φ ,then $\partial L/\partial\dot{\varphi} = \chi_0$ is a constant of motion ("Störmer's first integral").Denoting by p the particle's momentum and by ω the angle between its velocity and the φ direction,the following expression is obtained

$$\chi_0 = r\sin\vartheta\,(p\cos\omega + qA_\varphi) \qquad (2)$$

Because of the electric potential Ψ_0 , p is not conserved.However,the energies of interest here are considerably larger than the changes they undergo in the electric field so that,in order to simplify the calculation, p will be considered constant.From the example of part I it follows,in the limit of high conductivity and with a source of dipole moment M, that

in region II $$A_\varphi = \frac{3\mu_0 M}{8\pi R_0}\,\frac{\sin\vartheta}{r}$$

and in III $$A_\varphi = \frac{3\mu_0 M R_1}{8\pi R_0}\,\frac{\sin\vartheta}{r^2}$$

One notes that only the poloidal component of $\underline{B}$ contributes to (2). As in the treatment of motion in an ordinary dipole field (Störmer, 1950 ; Fermi, 1950) , all lengths will be measured in momentum-dependent "Störmer units"

$$l_{st.} = \left(\frac{3\,q\,M\mu_0 R_1}{8\pi R_0 p}\right)^{1/2} = (R_0 R_1)^{1/2}\left(\frac{qB}{p}\,R_0\right)^{1/2} \tag{3}$$

The first factor on the right is a mean value of r in region II ; the second one is the ratio between R_0 and the radius of gyration obtained, neglecting the toroidal component, on the inner boundary of region II at its its intersection with the equatorial plane. Choosing $B_S = 1$ gauss , $q = 1.6\ 10^{-19}$ coulomb, $R_0 = 7\ 10^{10}$ cm (the solar radius) and measuring p in Bev/c

$$l_{st} = 145\ (R_0 R_1/p)^{1/2} \tag{4}$$

Measuring all lengths in Störmer units and defining

$$\chi_1 = \chi_0\left(\frac{8\pi R_0}{3\mu_0 M R_1 q p}\right)^{1/2}$$

equation (2) becomes, in region II

$$\cos\omega = \frac{1}{r}\left(\frac{\chi_1}{\sin\vartheta} - \frac{\sin\vartheta}{R_1}\right) \tag{5 a}$$

and in region III

$$\cos\omega = \frac{\chi_1}{r\sin\vartheta} - \frac{\sin\vartheta}{r^2} \tag{5b}$$

Consider those particles with given momentum p and invariant χ_1. There will in general exist certain parts of the (r, ϑ) plane where equations (5) yield $|\cos\omega| > 1$ and which therefore are not accessible to these particles. The rest of the plane forms the "allowed region" , and if this region is multiply connected, trapping may occur.

The allowed region in region II is simply connected. To see this, it is best to consider a single radial direction with fixed ϑ. For all points having this ϑ, the bracketed term in (5a) is constant and $|\cos\omega|$ is a monotonic single valued function of r . As the origin is approached, $|\cos\omega|$ increases steadily until, for any ϑ, a forbidden region is reached (except for $\vartheta = \arcsin\,(\chi_1 R_1)^{\frac{1}{2}}$ where the allowed region extends to the origin). Thus the forbidden regions cluster around the origin and the allowed region is simply connected.

The allowed region of the dipole field in III, on the other hand, is multiply connected (Störmer, 1950 ; Fermi, 1950) if $\chi_1 > 2$. This region - with which the allowed region of II merges smoothly - consists then of an inner "trapped" region in which $r < 1$ everywhere and a "free" region where $r > 1$. Region III will include part of the trapped region only if $R_1 < 1$ or, by (4)

$$R_1 / R_0 \quad < \quad 21\,000/p \tag{6}$$

Assume that (6) holds and consider orbits in the equatorial plane of region II. As a simplifying assumption, every orbit with $\chi_1 > 2$ will then be considered trapped and every one with $\chi_1 < 2$ free. If (6) is just barely satisfied (e.g. $R_1 = 0.9$) it is easily seen from (5a) that no trapped orbits penetrate very far into region II. As R_1 decreases, this situation changes rapidly until at $R_1 = \frac{1}{2}$, for <u>any</u> r , orbits with $\omega < \pi/2$ are trapped and those with $\omega > \pi/2$ free. This is obviously when the anisotropy is most pronounced.

Assuming that the daily variation indeed arises in this fashion and that it is most pronounced at $p = 15$ Bev/c , (4) gives

$$R_1 = 350 R_o$$

If the solar radius is chosen for R_o , R_1 is approximately two astronomical units - considerably less than is generally believed,but not impossible (for discussion and references,see analysis by Axford,Dessler and Gottlieb [1963]).

There are two fundamental difficulties with this explanation.First,the polar field of the sun was observed to reverse its direction during the solar maximum of 1958 (Babcock,1959) while the cosmic ray anisotropy maintained its direction.It has been suggested that the sun's polar field is not the main source of the interplanetary magnetic field and that the latter does not reverse (Elliot,1960).In any case,it is hoped that this point will be resolved by future observations.

The second difficulty is that according to this explanation,the anisotropy occurs only in a very narrow energy band ; it does not explain,for instance, the observation of the daily variation underground (Regener,1962).It is possible,however,that a more realistic (and less abrupt) model of the outer boundary will resolve this problem.

The Density Gradient model

This theory has been described by Dattner and Venkatesan (1959) and was worked out in detail by Elliot (1960,1962).One of its basic assumptions is that the interplanetary magnetic field in the vicinity of the earth is perpendicular to the ecliptic ; of course,this does not agree with the radial stretching of magnetic lines of force by the solar wind,but this point will not be considered now.Let r be the distance from the sun to an observer on earth and let the discussion be restricted to particles with momentum p, which will have a gyration radius $a(r)$ in the earth's vicinity. Particles arriving tangentially to the earth's orbit from one direction will then have their guiding center at distance $(r + a)$, while those arriving from the opposite direction will have it at $(r - a)$. If there is a sunward gradient in the flux density Φ (reckoned at the guiding center of the particles it describes) the fluxes in the two directions are not equal and their ratio to the first order in (a/r) is $(1 + \delta)$, where (Elliot, 1960)

$$\delta = \frac{2a}{\Phi} \frac{d\Phi}{dr} \qquad (7)$$

There is good reason to believe a density gradient actually exists in interplanetary space, since it has been observed that the flux density arriving at earth undergoes a modulation connected with the solar cycle, and this modulation presumably extends only a finite distance from the sun. A different question is whether the gradient is pronounced in the vicinity of the earth's orbit. No evidence of an appreciable gradient was found by either Pioneer V (Simpson, Fan and Meyer, 1962) or Mariner II (Anderson, 1963); however, the radiation detectors aboard both these space probes were sensitive down to energies below 100 Mev, so that the absence of a density gradient in the energy range in which an anisotropy is observed on earth may not be considered proven.

A gradient of flux density is not, however, sufficient to create an anisotropy. As a simple illustration, suppose the radiation is acted upon by an electric field due to a positively charged sun. In such a field there will exist a flux density gradient, but because of Liouville's theorem, if the radiation is isotropic far from earth it will remain so anywhere in the field (effects of trapping are not considered now). It is instructive to examine the mechanism by which this happens.

Consider monoenergetic particles with charge q moving in the symmetry plane of a magnetic dipole field set up around the (positively charged) sun, and assume for simplicity that the motion is nonrelativistic. By Liouville's theorem, with phase-space density τ

$$\Phi = \frac{\tau}{m} p^3$$

$$\frac{1}{\Phi}\frac{d\Phi}{dr} = \frac{3}{p}\frac{dp}{dr}$$

Let W(r) be the mean kinetic energy at distance r and E(r) the (radial) electric field intensity there. Then

$$\frac{dp}{dr} = \frac{m}{p}\frac{dW}{dr} = -\frac{m}{p} q E$$

substituting a = p/qB one obtains

$$\delta = -6E/vB \qquad (8)$$

On the other hand, the electric field also causes the guiding center to drift in the direction of the anisotropy with velocity v_D, which by the nonrelativistic guiding center theory is

$$v_D = \frac{|\underline{E} \times \underline{B}|}{B^2} = \frac{E}{B} \qquad (9)$$

In the reference frame of its guiding center, a particle spends equal time moving in any direction in its plane of gyration. Given a large number of particles arriving from infinity, an observer moving with this frame sees an isotropic flux. The flux distribution in a frame of reference moving with velocity v_D relative to a frame of reference in which particles arrive isotropically has been calculated (for the extreme relativistic limit) by Compton and Getting (1935). For non-relativistic motion in which v_D is much smaller than the particle velocity v, one finds that the flux in the forward direction increases by a factor $1 + 3(v_D/v)$ while in the backward direction it is diminished by an equal amount. An anisotropy ratio $1 + (6E/vB)$ will therefore arise, completely cancelling out the gradient effect.

More generally, if a density gradient is responsible for the anisotropy, it cannot be caused by a simple potential field, e.g. by Ψ_o in the model used here. This is expected to hold even for relativistic particles, for Liouville's theorem remains true at relativistic velocities. It is of course possible that there may exist a nonconservative field in the solar system, by which particles gain (Warwick, 1962) or lose (Singer, Laster and Lencheck, 1962) energy. Such a field could, in principle, explain the anisotropy, were it not for the radial stretching of the lines of force. In any case, the solar cycle modulation and any flux density gradient which might be observed in space may very well be due to a conservative mechanism and have no connection with anisotropies.

Anisotropy due to the Compton-Getting effect

A theory has been developed by Ahluwalia and Dessler (1962) ascribing the anisotropy to relative motion between the earth and a frame of reference in which the cosmic radiation is isotropic. The orbital motion of the earth, for instance, would produce such an effect: this will however have an opposite phase to what is observed, and turns out (Dattner and Venkatesan, 1959) to have an amplitude of only 0.03%. In this theory, the sun is assumed to be surrounded by matter flowing radially outwards, as in the model used here. An electric field is then set up, which causes cosmic ray particles to drift across it and be isotropic in a frame of reference moving with the drift velocity. An earlier theory of this kind, by Brunberg and Dattner (1954), assumes the electric field is created by co-rotation of the interplanetary gas with the sun, extending at least to the earth's orbit.

In a highly conducting ionized gas an electric field will indeed exist, tending to the limiting value of $-[\underline{v} \times \underline{B}]$. However, if $\underline{B}$ is axisymmetric around the rotation axis

$$\mathrm{curl}\ \underline{E} = -\frac{\partial \underline{B}}{\partial t} = 0$$

so that $\underline{E}$ is conservative and according to the conclusions of the previous section, no anisotropy arises.

There remains the possibility that the field is not symmetric around the solar rotation axis, e.g. due to "beams" of enhanced velocity as suggested by Alfven (1956). In that case, however, it is hard to explain the constancy of the direction of the anisotropy. Assume the field is increasing at a certain time, creating an anisotropy in the observed direction. Several days later the field will be dropping to its previous value, so that particles in those orbits in which acceleration took place in the first instance will now be decelerated. One would then expect the anisotropy to reverse or at least undergo a considerable change in direction. One would also expect a much better correlation than is observed between the amplitude of the anisotropy and solar disturbances.

Conclusion

It will be seen from the preceding that all three explanations of the cosmic ray anisotropy meet with serious difficulties. While no detailed description should be expected from the crude model used, the following general conclusions may be drawn :

(1) Anisotropies due to trapped orbits are possible in the stretched dipole field, but only in a narrow energy range depending on the strength of the field's source and on the distance at which the lines of force begin closing. Of course, no anisotropy will be observed unless a preferential loss mechanism for trapped orbits exists .

(2) The existence - or lack of existence - of a radial flux density gradient in interplanetary space may be totally unrelated to the observed anisotropy.

(3) A conservative electric field, as proposed by Dessler and Ahluwalia, will not give rise to an anisotropy.

Acknowledgements

I wish to thank all the people with whom I discussed various points of this work, in particular Dr. K. G. McCracken, Dr. E. Ray and Dr. Frank Jones.

The major part of this work was performed under a National Academy of Sciences post-doctoral resident research associateship.

REFERENCES

A. S. Ahluwalia and A. J. Dessler: "Diurnal Variation of Cosmic Radiation Intensity produced by a Solar Wind," Planetary and Space Science 9, 195 (1962).

H. Alfven: "Solar Magnetic Field and Diurnal Variation of Cosmic Radiation," Physical Review 72, 88 (1947).

H. Alfven: "The Sun's General Magnetic Field," Tellus 8, 1 (1956).

H. R. Anderson, 44th annual meeting of the AGU, abstract P-59; Transactions of the American Geophysical Union 44, 79 (1963).

W. I. Axford, A. J. Dessler and B. Gottlieb: "Termination of Solar Wind and Solar Magnetic Field," Astrophysical Journal 137, 1268 (1963).

Harold D. Babcock: "The Sun's Polar Magnetic Field," Astrophysical Journal 130, 364 (1959).

E. A. Brunberg and A. Dattner: "On the interpretation of the Diurnal Variation of Cosmic Rays," Tellus 6, 73 (1954).

A. H. Compton and I. A. Getting: "An apparent effect of galactic rotation on the intensity of Cosmic Rays," Physical Review 47, 817 (1935).

A. Dattner and D. Venkatesan: "Anisotropies in Cosmic Radiation," Tellus 11, 239 (1959).

Kirby Dwight: "Solar Magnetic Moment and Diurnal Variation in Intensity of Cosmic Radiation," Physical Review 78, 40 (1950).

H. Elliot: "Cosmic Ray Intensity Variations and the Interplanetary Magnetic Field," Philosophical Magazine 5, 601 (1960).

H. Elliot: "Modulation of the Cosmic Ray Intensity by the Interplanetary Magnetic Field," Journal of the Physical Society of Japan 17, supplement A-II, 588 (1962).

Enrico Fermi: "Nuclear Physics" (notes compiled by J. Orear, A. H. Rosenfeld and R. A. Schluter) University of Chicago Press, Revised Edition 1950.

Ludwig Janossy: "Uber Einem Möglichen Einflus des Magnetfeldes der Sonne auf die in Erdnähe beobachtete Höhenstrahlung," Zeitschrift für Physik 104, 430 (1937).

K. G. McCracken, U. R. Rao and D. Venkatesan: "Asymptotic Cones of Acceptance and their use in the study of the Daily Variation of Cosmic Radiation," Journal of Geophysical Research 68, 345 (1963).

Victor H. Regener: "Solar Diurnal Variation of Cosmic Rays Underground Near the Geomagnetic Equator", Journal of the Physical Society of Japan 17, Supplement A-II, 481 (1962).

J. A. Simpson, C. Y. Fan and Peter Mayer: "The Cosmic Ray Intensity Gradient in Space during Solar Modulation," Journal of the Physical Society of Japan 17, supplement A-II, 505 (1962).

S. F. Singer, H. Laster and A. M. Lencheck: "Forbush Decreases Produced by Diffusing Deceleration Mechanism in Interplanetary Space," Journal of the Physical Society of Japan 17, Supplement A-II, 583 (1962).

Carl Störmer: "The Polar Aurora," Oxford 1955.

C. S. Warwick: "Propagation of Solar Particles and the Interplanetary Magnetic Field," Journal of Geophysical Research 67, 1333 (1962).